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# Mathematical Modeling of Quasi Static Thermoelastic Transient behavior of thick circular plate with Internal Heat Generation

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**ABSTRACT**
The present paper deals with the determination of displacement and thermal transient stresses in a thick circular plate with internal heat generation. External arbitrary heat supply is applied at the upper surface of a thick circular plate, whereas the lower surface of a thick circular plate is insulated and heat is dissipated due to convection in surrounding through lateral surface. Here we compute the effects of internal heat generation of a thick circular plate in terms of stresses along radial direction. The governing heat conduction equation has been solved by using integral transform method. The results are obtained in series form in terms of Bessel's functions and the results for temperature change and stresses have been computed numerically and illustrated graphically.
***Keywords*** Thermal stresses, internal heat generation, quasi static.

## I. Introduction

Non isothermal problems of the theory of elasticity became increasingly important. This is due to their wide applications in diverse fields. Nowacki [1] has determined the temperature distribution on the upper face, with zero temperature on the lower face and the circular edge thermally insulated. Sharma et al. [2] studied the behavior of thermoelastic thick plate under lateral loads. Kulkarni and Deshmukh [3] studied the thermoelastic behavior of thick circular plate with the help of arbitrary initial heat supply on the upper surface. Kedar and Deshmukh [4] has determined the quasi-static thermal stresses due to an instantaneous point heat source in a circular plate subjected to time dependent heat flux at the fixed circular boundary. Most recently Bhongade and Durge [5] considered thick circular plate and discuss, effect of internal heat generation on steady state behavior of thick circular plate.

In this paper thick circular plate is considered and discussed its thermoelasticity with the help of the Goodier's thermoelastic displacement potential function and the Michell's function. To obtain the temperature distribution integral transform method is applied. The results are obtained in series form in terms of Bessel's functions and the temperature change, displacement function and stresses have been computed numerically and illustrated graphically. Here we compute the effects of internal heat generation in terms of stresses along radial direction. A mathematical model has been constructed of a thick circular plate with the help of numerical illustration by considering copper (pure) circular plate. No one previously studied such type of problem. This is new contribution to the field.

The direct problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines and the role of rolling mill, base of furnace of boiler of a thermal power plant, gas power plant.

## II. Formulation of the Problem

Consider a thick circular plate of radius $a$ and thickness $h$ defined by $0 \le r \le a, \frac{-h}{2} \le z \le \frac{h}{2}$. Let the plate be subjected to external arbitrary heat supply is applied on the upper surface, the lower surface is insulated and heat convection is maintained at fixed circular edge $(r = a)$. Assume the circular boundary of a thick circular plate is free from traction. Under these prescribed conditions, the quasi-static thermal transient stresses, temperature and displacement in a thick circular plate with internal heat generation are required to be determined.

The differential equation governing the displacement potential function $\phi(r, z, t)$ is given by,

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = K\tau \qquad (1)$$

where $K$ is the restraint coefficient and temperature change $\tau = T - T_i$, $T_i$ is initial temperature. Displacement function $\phi$ is known as Goodier's thermoelastic displacement potential.

Temperature of the plate at time $t$ satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

with the boundary conditions

$$T = 0 \qquad at\ t = 0,\ 0 \le r \le a \tag{3}$$

$$q(r,z,t) = 0 \quad at\ t = 0, \tag{4}$$

$$T = f(r,t) \quad at\ z = \frac{h}{2},\ 0 \le r \le a \tag{5}$$

$$\frac{\partial T}{\partial Z} = 0 \qquad at\ z = -\frac{h}{2},\ 0 \le r \le a \tag{6}$$

$$h_1 T + \frac{\partial T}{\partial r} = 0\ at\ r = a \tag{7}$$

where $\alpha$ is the thermal diffusivity of the material of the plate, $k$ is the thermal conductivity of the material of the plate, $q$ is the internal heat generation and $h_1$ is heat transfer coefficient.

The Michell's function *M* must satisfy

$$\nabla^2\nabla^2 M = 0 \tag{8}$$

where,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{9}$$

The components of the stresses are represented by the thermoelastic displacement potential $\phi$ and Michell's function *M* as

$$\sigma_{rr} = 2G\left\{\frac{\partial^2\phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right]\right\} \tag{10}$$

$$\sigma_{\theta\theta} = 2G\left\{\frac{1}{r}\frac{\partial\phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right]\right\} \tag{11}$$

$$\sigma_{zz} = 2G\left\{\frac{\partial^2\phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left[(2-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{12}$$

$$and\ \sigma_{rz} = 2G\left\{\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial r}\left[(1-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{13}$$

where *G* and *v* are the shear modulus and Poisson's ratio respectively.
The boundary conditions on the traction free surfaces of circular plate are

$$\sigma_{rr} = \sigma_{rz} = 0 \ \text{ at } \ r = a \tag{14}$$

Equations (1) to (14) constitute mathematical formulation of the problem.

## III. Solution of the Heat Conduction Equation

To obtain the expression for temperature $T(r, z, t)$, we introduce the finite Hankel transform over the variable $r$ and its inverse transform are

$$\bar{T}(\beta_m, z, t) = \int_{r'=0}^{a} r' K_0(\beta_m, r')\ T(r', z, t)\, dr' \tag{15}$$

$$T(r,z,t) = \sum_{m=1}^{\infty} K_0(\beta_m, r)\ \bar{T}(\beta_m, z, t) \tag{16}$$

where, $$K_0(\beta_m, r) = \frac{\sqrt{2}}{a}\frac{\beta_m}{({h_1}^2 + {\beta_m}^2)^{\frac{1}{2}}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)} \tag{17}$$

and $\beta_1, \beta_2, \ldots$ are the positive roots of the transcendental equation

$$\beta_m\ J_0{}'(\beta_m a) + h_1 J_0(\beta_m a) = 0 \tag{18}$$

where $J_n(x)$ is the Bessel function of the first kind of order n. The Hankel transform-H, defined in Eq. (15), satisfies the relation

$$H\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] = -{\beta_m}^2\bar{T}(\beta_m, z, t) \tag{19}$$

On applying the finite Hankel transform defined in the Eq. (15) and its inverse transform defined in Eq. (16) to the Eq. (2), one obtains the expression for temperature as

$$T(r,z,t) = \sum_{m=1}^{\infty}\left\{\left(\frac{\sqrt{2}}{a}\frac{\beta_m}{(h_1{}^2+\beta_m{}^2)^{\frac{1}{2}}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}\right) \times \sum_{n=1}^{\infty}\left[\begin{array}{c} \frac{(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2} - f_1(t)\right) \\ +\frac{2\alpha}{(-1)^n\ h}\ sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] f_2(t) + C_1(\beta_m, z\ ,t) \end{array}\right]\right\} \tag{20}$$

where,

$C_1(\beta_m, z\ ,t) = L^{-1}[A_1(\beta_m, z\ ,s)]$, $A_1(\beta_m, z\ ,t)$ is particular integral of differential equation,

$$f_1(t) = \int_0^t A_1(\beta_m,\frac{h}{2}, t-u)e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]u}du$$

and

$$f_2(t) = -\int_0^t A_1(\beta_m,-\frac{h}{2}, t-u)e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]u}du$$

Michell's Function $M$

We select $M$ which satisfy Eq. (8) is given by

$$M = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[B_{mn}\ J_0(\beta_m r) + C_{mn}\ \beta_m\ r\, J_1(\beta_m r)]\, cosh[\beta_m(z+\frac{h}{2})] \tag{21}$$

where $B_{mn}$ and $C_{mn}$ are arbitrary functions, which can be determined by using condition (14).

**Goodiers Thermoelastic Displacement Potential and Thermal Stresses**

Displacement function $\phi(r,z,t)$ which satisfies Eq. (1) as

$$\phi(r,z,t) = \sum_{m=1}^{\infty}\left[\frac{\sqrt{2}}{a}\frac{K\,\beta_m}{(h_1{}^2+\beta_m{}^2)^{\frac{1}{2}}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}\right] \times \sum_{n=1}^{\infty}\left[\begin{array}{c} \frac{-(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \times\left(\frac{-F(\beta_m,\ t)\ [e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2} - f_1(t)\right) \\ -\frac{2\alpha}{(-1)^n\ h} sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \\ -\frac{(2n-1)\pi\alpha\ f_2(t)e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}}{(-1)^n h^2\sqrt{K+\beta_m{}^2}\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \end{array}\right] \tag{22}$$

Now using Eqs. (20), (21) and (22) in Eq. (10), (11), (12) and (13), one obtains the expressions for stresses respectively as

For convenience setting

$$g_1(r) = \left[\frac{-J_1{}'(\beta_m r)\ \beta_m}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} + J_0(\beta_m r)\right],$$

$$g_2(r) = \left[\frac{-J_1(\beta_m r)\,\beta_m}{r\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} + J_0(\beta_m r)\right],$$

$$A = \left[\frac{-J_1{}'(\beta_m a)\ \beta_m{}^2}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} + J_0(\beta_m a)\right]$$

and $R = \left\{\begin{array}{l}\left[\ (2v-1)\ \beta_m{}^3 J_0(\beta_m a) +\ a\, J_0(\beta_m a)\beta_m{}^4\ \right]\ J_1(\beta_m a)\ \beta_m{}^3 \\ -\left[\ a\, J_0(\beta_m a)\beta_m{}^4 -\ 2(1-v)\ \beta_m{}^3 J_1(\beta_m a)\right]\ J_1{}'(\beta_m a)\,\beta_m{}^3\end{array}\right\}$

$$\frac{\sigma_{rr}}{K} = 2G\sum_{m=1}^{\infty}\left\{\left[-\frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{1}{J_0(\beta_m a)}\right]\right.$$
$$\times\sum_{n=1}^{\infty}\left(\left[\begin{array}{c}\frac{(2n-1)\alpha\pi}{(-1)^n h^2}\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\\ \times\left(\frac{-F(\beta_m,t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right)\\ +\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\ f_2(t)\end{array}\right]\times g_1(r)\right.$$
$$\left.+\frac{(2n-1)\pi\,\alpha}{(-1)^n h^2}\frac{\beta_m{}^2 J_1{}'(\beta_m r)\,f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\frac{e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}}{\sqrt{K+\beta_m{}^2}}+J_0(\beta_m r)\,C_1(\beta_m,z,t)\right)$$
$$\left.+\sum_{m=1}^{\infty}\left[\begin{array}{c}B_{mn}\ J_1{}'(\beta_m r)\\ +C_{mn}\ \langle\beta_m r\,J_1(\beta_m r)+(2v-1)J_0(\beta_m r)\rangle\end{array}\right]\beta_m{}^3\sinh\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\} \tag{23}$$

$$\frac{\sigma_{\theta\theta}}{K} = 2G\sum_{m=1}^{\infty}\left\{\left[-\frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{1}{J_0(\beta_m a)}\right]\right.$$
$$\times\sum_{n=1}^{\infty}\left(\left[\begin{array}{c}\frac{(2n-1)\alpha\pi}{(-1)^n h^2}\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\\ \times\left(\frac{-F(\beta_m,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right)\\ +\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\ f_2(t)\end{array}\right]\times g_2(r)\right.$$
$$\left.+\frac{(2n-1)\pi\,\alpha}{(-1)^n h^2}\frac{\beta_m{}^2 J_1{}'(\beta_m r)\,f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\frac{e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}}{\sqrt{K+\beta_m{}^2}}+J_0(\beta_m r)\,C_1(\beta_m,z,t)\right)$$
$$\left.+\sum_{m=1}^{\infty}\left[\ C_{mn}\ \ \beta_m{}^3\ J_0(\beta_m r)\ +\ B_{mn}\ \frac{\beta_m{}^2}{r}J_1(\beta_m r)\right]\sinh\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\} \tag{24}$$

$$\frac{\sigma_{zz}}{K} = 2G\sum_{m=1}^{\infty}\left\{\left[\frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}\right]\right.$$
$$\times\sum_{n=1}^{\infty}\left(\left[\begin{array}{c}\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\left[\frac{(2n-1)^2\pi^2}{4h^2\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}-1\right]\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\\ \times\left(\frac{-F(\beta_m,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right)\\ +\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\ f_2(t)\end{array}\right]\right.$$
$$\left.+\frac{(2n-1)\pi\,\alpha}{(-1)^n h^2}\frac{\sqrt{K+\beta_m{}^2}\,f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}+\ C_1(\beta_m,z,t)\right)$$
$$\left.+\left[\begin{array}{c}C_{mn}\ \langle 2(1-v)+\beta_m\ r\,J_1(\beta_m r)\rangle\\ -B_{mn}\ J_0(\beta_m r)\end{array}\right]\beta_m{}^3\sinh\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\} \tag{25}$$

$$\frac{\sigma_{rz}}{K} = 2G\left\{\sum_{m=1}^{\infty}\left[\frac{-\sqrt{2}}{a}\frac{\beta_m{}^2}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{J_1(\beta_m r)}{J_0(\beta_m a)}\right]\right.$$

$$\times\sum_{n=1}^{\infty}\left(\left[\begin{array}{c}\frac{(2n-1)^2\pi^2\alpha}{2(-1)^n h^3}\ sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m\ ,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right) \\ -\frac{(2n-1)\alpha\pi}{(-1)^n h^2}cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\end{array}\right] \\ +\frac{(2n-1)\pi\ \alpha}{(-1)^n h^2}\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}\right)$$

$$\left.+\sum_{m=1}^{\infty}\left[\begin{array}{c}C_{mn}\ \langle\ (2v-2)\,J_1(\beta_m r)+r\,\beta_m\,J_0(\beta_m r)\ \rangle \\ +B_{mn}\ J_1(\beta_m r)\end{array}\right]\beta_m{}^3cosh\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\} \tag{26}$$

In order to satisfy condition (14), solving Eqs. (23) and (26) for $B_{mn}$ and $C_{mn}$ , one obtains

$$B_{mn} = \sum_{m=1}^{\infty}\left(\frac{\sqrt{2}}{R\ a}\frac{K\,\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{1}{J_0(\beta_m a)}\right)\left\{\frac{[\ (2v-2)J_1(\beta_m a)+a\ \beta_m J_0(\beta_m a)]}{sin\,h[\beta_m(z+\frac{h}{2})]}\right.$$

$$\times\sum_{n=1}^{\infty}\left(\left[\begin{array}{c}\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\ cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m\ ,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right) \\ +\frac{2\alpha}{(-1)^n\ h}sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\ f_2(t)\end{array}\right]\times A \\ +\frac{(2n-1)\pi\ \alpha}{(-1)^n h^2}\frac{\beta_m{}^2\,J_1{}'(\beta_m a)\,f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\frac{e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}}{\sqrt{K+\beta_m{}^2}}+J_0(\beta_m a)\ C_1(\beta_m,z\ ,t)\right)$$

$$-\frac{\beta_m\ J_1(\beta_m a)}{J_0(\beta_m a)}\frac{\left[(2v-1)\ \beta_m{}^3J_0(\beta_m a)+a\,J_1(\beta_m a)\beta_m{}^4\right]}{cos\,h[\beta_m(z+\frac{h}{2})]}$$

$$\left.\times\sum_{n=1}^{\infty}\left[\begin{array}{c}\frac{(2n-1)^2\pi^2\alpha}{2(-1)^n h^3}sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\times\left(\frac{-F(\beta_m,t\,)[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}-f_1(t)\right) \\ -\frac{(2n-1)\pi\alpha}{(-1)^n h^2}cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \\ +\frac{(2n-1)\pi\alpha\ f_2(t)e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}}{(-1)^n h^2\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\end{array}\right]\right\} \tag{27}$$

$$C_{mn} = \sum_{m=1}^{\infty}\left(\frac{\sqrt{2}}{a\ R}\frac{K\,\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{1}{J_0(\beta_m a)}\right)\left\{\frac{\boldsymbol{\beta_m{}^3 J_1(\beta_m a)}}{\boldsymbol{sinh[\beta_m(z+\frac{h}{2})]}}\right.$$

$$
\left.\begin{aligned}
&\times \sum_{n=1}^{\infty}\left(\begin{aligned}&\left[\begin{aligned}&\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\\
&\times\left(\frac{-F(\beta_m,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right)\\
&+\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] f_2(t)\end{aligned}\right]\times A\\
&+\frac{(2n-1)\pi\,\alpha}{(-1)^n h^2}\,\frac{\beta_m{}^2 J_1{}'(\beta_m a)\, f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\,\frac{e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}}{\sqrt{K+\beta_m{}^2}}+J_0(\beta_m a)\, C_1(\beta_m, z, t)\end{aligned}\right)\\
&-\frac{J_1{}'(\beta_m a)\, J_1(\beta_m a)\ \beta_m{}^4}{\cos h[\beta_m(z+\frac{h}{2})]}\\
&\times \sum_{n=1}^{\infty}\left[\begin{aligned}&\frac{(2n-1)^2\pi^2\alpha}{2(-1)^n h^3}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\times\left(\frac{-F(\beta_m,t)\ \left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}-f_1(t)\right)\\
&-\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\\
&+\frac{(2n-1)\pi\alpha\ f_2(t) e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}}{(-1)^n 2h^2\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\end{aligned}\right]
\end{aligned}\right\} \quad (28)
$$

## 4. Special case and Numerical Calculations

Setting

(1) $f(r)=\delta(r-r_1)e^{-t},\ r_1\ =0.5m, a=1m.$

where $\delta(r)$ is well known dirac delta function of argument r.

$$F(\beta_m)=\frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{\beta_m{}^2+h_1{}^2}}\frac{r_1\, J_0(\beta_m r_1)}{J_0(\beta_m a)}e^{-t}$$

(2) $q(r,z,t)=\ \delta(r-r_0)\ t\cos(\frac{2\pi z}{h})\ ,\ r_0\ =0.5m, h=0.25m$

$$\bar{q}\,(\beta_m,z,t)=\ \frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{\beta_m{}^2+h_1{}^2}}\ \frac{\cos\left(\frac{2\pi z}{h}\right) r_0\, J_0(\beta_m r_0)\ t}{J_0(\beta_m a)}$$

$$\bar{\bar{q}}\ (\beta_m,z,s)=\ \frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{\beta_m{}^2+h_1{}^2}}\ \frac{\cos\left(\frac{2\pi z}{h}\right) r_0\, J_0(\beta_m r_0)}{J_0(\beta_m a)}\frac{1}{s^2}$$

Material Properties

The numerical calculation has been carried out for a copper (pure) circular plate with the material properties defined as,

Thermal diffusivity $\alpha$ = 112.34× $10^{-6}\ m^2 s^{-1}$,

Specific heat $c_\rho\ =383J/Kg,$

Thermal conductivity $k$ = 386 W/m K,

Shear modulus $G=48\ G\ pa,$

Poisson ratio $\vartheta=0.3.$

Roots of Transcendental Equation

The $\beta_1=1.7887,\ \beta_2=4.4634, \beta_3=7.4103,\ \beta_4=10.4566,\ \beta_5=13.543, \beta_6=16.64994$ are the roots of transcendental equation $\beta_m\, J_0{}'(\beta_m a)\ +\ h_1\, J_0(\beta_{m,} a)=0$. The numerical calculation and the graph has been carried out with the help of mathematical software Matlab.

## IV. Discussion

In this paper a thick circular plate is considered and determined the expressions for temperature, displacement and stresses due to internal heat generation within it and we compute the effects of internal heat generation in terms of stresses along radial direction. As a special case mathematical model is constructed for considering copper (pure) circular plate with the material properties specified above.

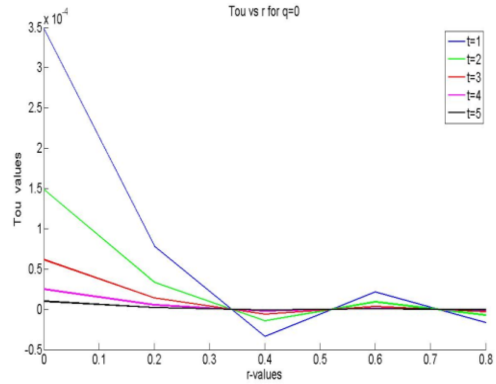


**Fig. 1** Temperature distribution $(q = 0)$

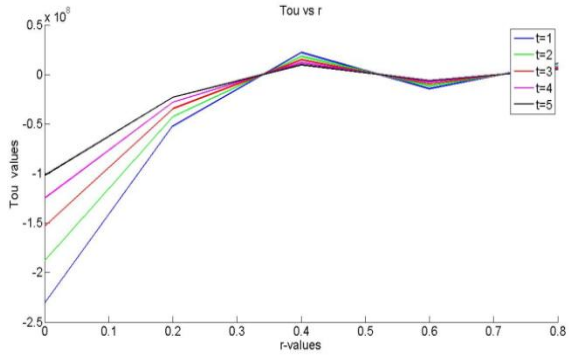


**Fig. 2** Temperature distribution $( q \neq 0)$

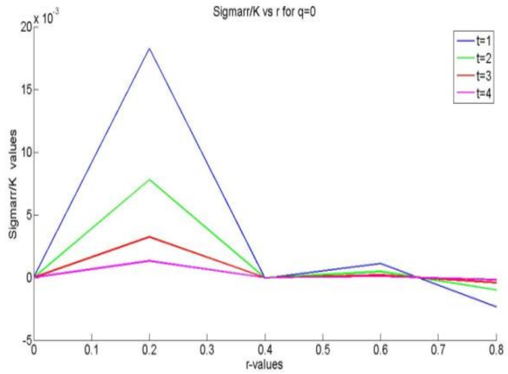


**Fig. 3** Radial stress function $\frac{\sigma_{rr}}{K} (q = 0)$

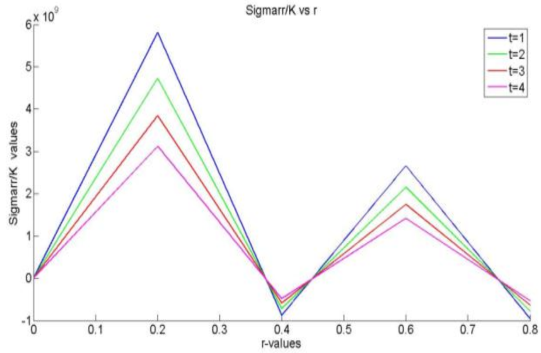


**Fig.4**Radial stress function $\frac{\sigma_{rr}}{K} (q \neq 0)$

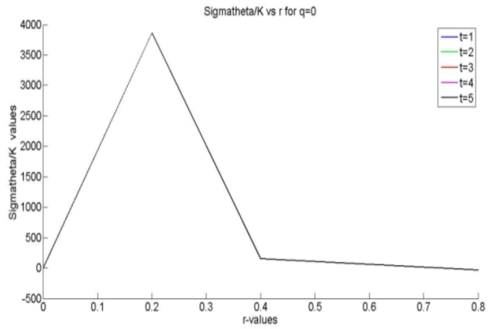


**Fig. 5** Angular stress function $\frac{\sigma_{\theta\theta}}{K} (q = 0)$

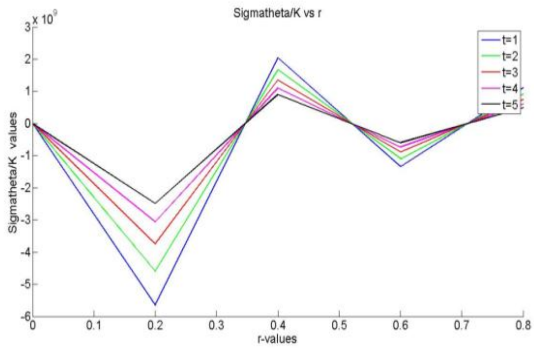


**Fig. 6** Angular stress function $\frac{\sigma_{\theta\theta}}{K} (q \neq 0)$

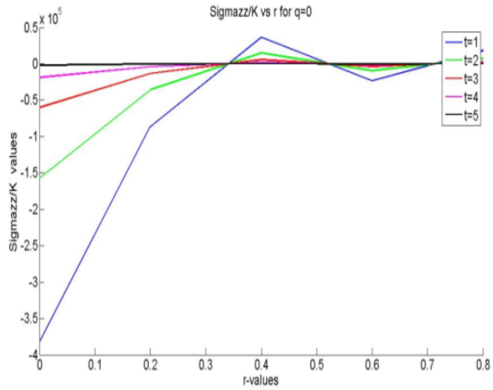


**Fig.7** Axial stress function $\frac{\sigma_{zz}}{K} (q = 0)$

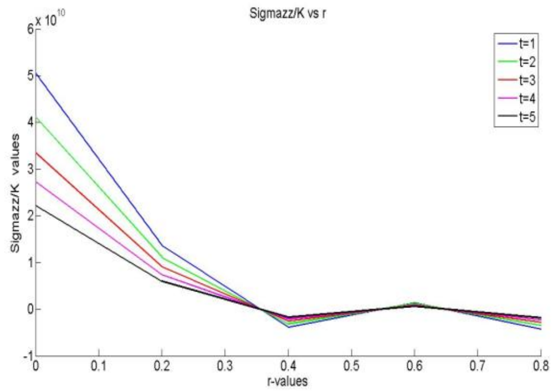


**Fig. 8** Axial stress function $\frac{\sigma_{zz}}{K} (q \neq 0)$

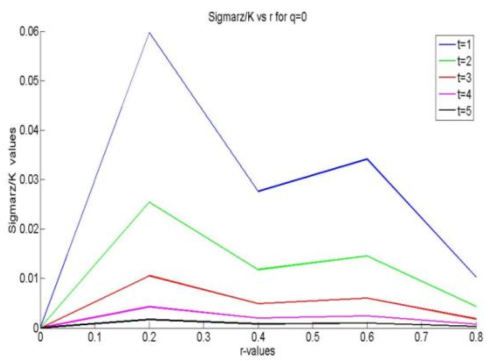


**Fig.9** Stress function $\frac{\sigma_{rz}}{K}(q = 0)$

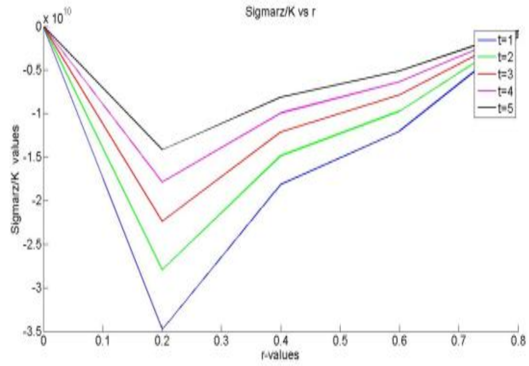


**Fig. 10** Stress function $\frac{\sigma_{rz}}{K}(q \neq 0)$

From Fig. 1 and 2, it is observed that the temperature vary with internal heat generation.
From Fig. 3 and 4, it is observed that the radial stresses $\frac{\sigma_{rr}}{K}$ vary along radial direction due to internal heat generation, it develops compressive stress in the radial direction.
From Fig. 5 and 6, it is observed that the angular stresses $\frac{\sigma_{\theta\theta}}{K}$ vary along radial direction due to internal heat generation, it develops tensile stress in the radial direction.
From Fig. 7 and 8, it is observed that the axial stresses $\frac{\sigma_{zz}}{K}$ vary along radial direction due to internal heat generation, it develops compressive stress in the radial direction.
From Fig. 9 and 10, it is observed that the stresses $\frac{\sigma_{rz}}{K}$ vary along radial direction due to internal heat generation.

## V. Conclusion

We can summarize that due to internal heat generation in thick circular plate the radial stress develops compressive stress, whereas the angular stress is tensile. Also, it can be observed from the figures that the value of temperature and stresses due to internal heat generation are higher as compared to plate with no heat generation.

The results obtained here are useful in engineering problems particularly in the determination of state of stress in thick circular plate and base of furnace of boiler of a thermal power plant and gas power plant.

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